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МОДЕЛИРОВАНИЕ ВНУТРЕННИХ ВОЛН В БАЛТИЙСКОМ МОРЕ

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Дан краткий обзор недавних исследований внутренних волн в Балтийском море. Обсуждаются данные наблюдений внутренних волн, представлена также база данных фоновых гидрологических параметров, которые главным образом определяют характеристики внутренних волн. Эти наборы данных используются для выбора примеров путей распространения волн, которые содержат критические точки. Численное моделирование распространения внутренних уединенных волн осуществляется с использованием относительно простой модели, основанной на уравнении Гарднера и выбранной благодаря ее явным преимуществам. Эта модель позволяет проводить вычисления с высоким разрешением при умеренных компьютерных ресурсах и способна воспроизводить трансформацию внутренних волн в условиях горизонтально-неоднородной стратификации водной среды. Приводятся результаты расчетов трансформации внутренних солитонов в Балтийском море вдоль разрезов с реалистичными горизонтально-переменными гидрологическими условиями.

Ключевые слова: слабонелинейные внутренние волны, Балтийское море, уравнение Гарднера, неоднородная среда.

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MODELLING OF INTERNAL WAVES IN THE BALTIC SEA

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A brief review of recent studies of internal waves in the Baltic Sea is given. The data of various observations of internal waves in the Baltic region are discussed and a database for the background hydrological parameters that govern the basic appearance of the internal wave shape is introduced. These data sets are employed to select examples of pathways of wave propagation that contain critical points. Numerical modeling of propagation of internal solitary waves across the sea is carried out using the relatively simple model based on Gardner's equation. This model was chosen due to its clear advantages. It is robust, allows high-resolution calculations with modest computer resources, and is able to reproduce the transformation of internal waves in horizontally inhomogeneous sea. The results of calculations of the transformation of internal solitons in the Baltic Sea along such pathways in a realistic (spatially variable) set of hydrological conditions are presented.

Key words: internal waves, weakly nonlinear solitons, Baltic Sea, Gardner's equation, inhomogeneous medium.

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Introduction. The main mechanism for the generation of internal waves (IWs) in the ocean is the interaction of the barotropic tide with the uneven bottom. The Baltic Sea is a micro-tidal shallow shelf sea with predominant depths from 40 to 100 m [1]. However, even in such seas there are mechanisms that generate intense IWs. For example, IWs can be excited by various dynamic processes associated with the development and relaxation of coastal upwelling and downwelling, vortices of various scales, storm surge phenomena, oscillations of hydrological fronts, etc. Satellite observations of surface manifestations of IWs in seas without tides are summarized in refs. [2—5]. Packages of IWs in the Baltic Sea are clearly visible on satellite photographs on the Space Research Institute website (http://iki.rssi.ru/asp/iw_images/index.html#baltic).

The increasing number of wind power plants, oil platforms and cables and pipelines on the seabed in the Baltic Sea motivates us to take a fresh look at the problems of the redistribution of sediments and the dynamics of erosion near subsea parts of engineering structures. The influence of IWs substantially contributes to these processes that can lead to the loss of stability of engineering structures. As a consequence, such processes lead to an increase in the risk of disasters in the coastal zone and even more in the deeper areas of the Baltic Sea where IWs may serve as the primary driver of sediment dynamics. Therefore, the development of methods for modelling and analyzing IW regimes and for the estimates of the associated velocity and pressure fields becomes particularly relevant for the formulation of recommendations for ensuring safety and improving of engineering structures.

One of the subtasks here is the development of maps of kinematic and nonlinear characteristics (within the framework of weakly nonlinear models) of the field of IWs on various scales. Such maps play the same role as, for example, bathymetric maps for surface waves. This task was partially realized in [6]. In this case, the vertical structure of the IW field is determined by the solution of a linear boundary value problem. This structure is easily calculated numerically for each vertical density profile and horizontal shear flow. The velocity field induced by an IW can also be easily determined and its vertical structure becomes immediately visible in the linear approximation. An important feature of this structure is that the maximum horizontal velocities of fluid particles in the waves that represent the most energetic lowest mode are reached either at the bottom or near the sea surface, depending on the background conditions. This asymmetry, expressed quantitatively, makes it possible to understand the loads directly exerted by IWs on various structures as well as the degree of influence of propagating IWs on the mobilization and transport of sediment particles located in the near-bottom and near-surface layers.

To simplify the calculations, we created an electronic atlas (a set of maps and tools for selecting and working with various wave properties) of the kinematic and nonlinear characteristics (coefficients at the relevant terms in the underlying nonlinear equation) of IWs for the Baltic Sea on the basis of all possible sources of hydrological data. This atlas helps to estimate the horizontal variability of the IW field and the polarity, amplification and possible breaking of IWs (http://lmnad.nntu.ru/en/projects/igw_kinematic_atlas/, http://lmnad.nntu.ru/en/projects/igwresearch/).

In the present paper we start from a review of observations of the IW manifestations in the Baltic Sea (Section 1). To simulate the transformations of internal solitary waves (ISW) along certain pathways in the Baltic Sea (Section 2), we have used a generally accepted model that is based on Gardner's equation with variable coefficients. A detailed description of the model can be found, for example, in [7—9]. Although the Baltic Sea is situated in middle and rather high latitudes, these calculations do not take into account effects of the Earth's rotation. The relevant impact is negligible because the considered waves are relatively short in comparison with tidal and inertial waves.

1. Observations of internal waves in the Baltic Sea. Since the stratification of the Baltic Sea is relatively strong and persistent, IWs must be a common feature in this basin, even though the number of studies into such waves is relatively small [10]. Several kinds of IWs can exist in this water body because of the variety of forcing factors and the complexity of its bathymetry. The relevant field observations are scarce. The generation of IWs in the Baltic Sea is mainly explained by the impact of strong winds [11] and the radiation of barotropic inertial waves. When these waves propagate through stratified water on a sloping bottom, they generate baroclinic inertial waves that in turn cascade energy into small-scale IWs [12, 13].

Tidal oscillations of the Baltic Sea level are fairly small: from 4 cm (Klaipėda) up to 10 cm in some sections of the Gulf of Finland [1]. The associated current speed, however, cannot be neglected. It reaches about 10 cm/s in the middle of the Gulf of Finland [14] but still remains much below the intensity of motions that are apparently driven by large IWs in this basin.

Generation of IWs in microtidal seas is possible due to several other dynamic processes such as the development and relaxation of coastal upwelling, vortices of different scales, relaxation of storm surges, oscillations of hydrological fronts, etc. Several studies are devoted to in situ observations and numerical modelling of the generation and propagation of short-period IWs in microtidal and nontidal seas, based on experimental data obtained by contact measurements [15—17].

In recent years the number of remote sensing observations of surface manifestations of IWs has increased significantly, including observations for nontidal seas [2—5, 18]. These observational data suggest that:

1) although IWs in micro-tidal seas are less intense compared with IWs generated by the tidal flow on oceanic shelves, mechanisms of their origin are much more diverse;

2) significant seasonal and interannual variability of both the IW activity and its manifestations is evident in the Synthetic Aperture Radar (SAR) images of the sea surface;

3) a clear correlation exists between the frequency of occurrence of surface manifestations of IWs in SAR images and the pycnocline position: frequent occurrence of surface manifestations of IWs corresponds to a sharp and shallow pycnocline.

SAR observations of surface manifestations of IWs over the Baltic Sea area are quite difficult because of rapidly varying meteorological conditions. A number of factors such as intensification or weakening of wind (calm, windless regions), development of choppiness, rough sea, algal blooms, heavy precipitation, and passage of sharp atmospheric and wind fronts or the presence of atmospheric IWs undermine the identification of surface manifestations of IWs. The imprint of IWs can be masked by the processes in the near-water layer of the atmosphere [2]. Therefore it is not surprising that surface manifestations of IWs are relatively rare for the Baltic Sea and that only a few wave events were detected from satellite SAR images.

As an example, the main characteristics of five IW events reconstructed from SAR images are presented table and figs. 1, 2. The density and buoyancy profiles for the geographical locations of these observations (fig. 3) are evaluated from the monthly averaged long-term temperature and salinity fields calculated using the Rossby Centre Ocean circulation model [19—21]. Note that four of the five stratification profiles have quite a sharp and shallow pycnocline (5—10 m below the surface, while the total depth was 40—200 m).

The paper [5] reports 11 events of surface manifestations of IWs in the Baltic Sea proper and in the Gulf of Bothnia and 12 events in the Danish straits in 2009—2010. The IWs in the Danish straits are generated by

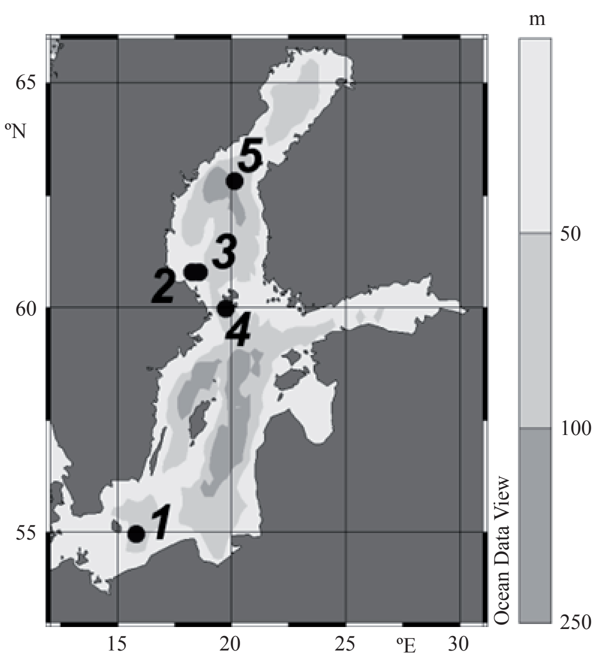

Fig. 1. Locations of observations 1—5 (table) of surface manifestations of IWs on satellite SAR images of the Baltic Sea on the background of bathymetry.

Рис. 1. Географические координаты местоположений наблюдений, приведенных в таблице, поверхностных проявлений внутренних волн по данным спутниковых снимков Балтийского моря. Точки даны на фоне батиметрической карты.

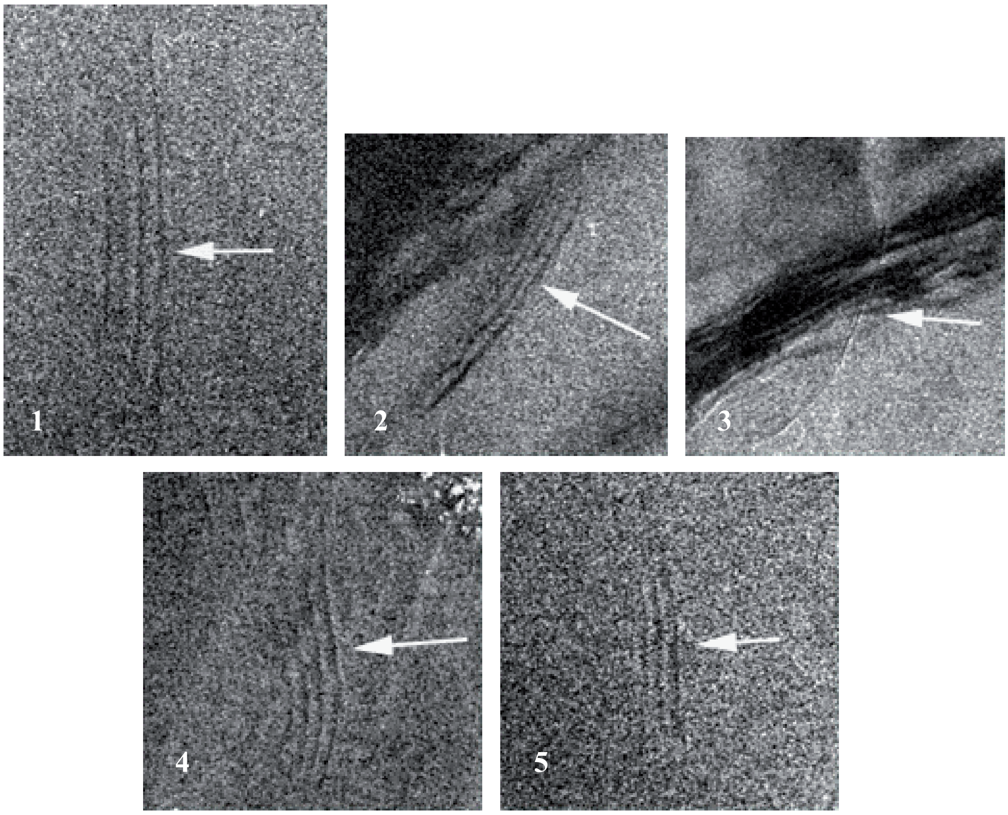

Fig. 2. Surface manifestations of IWs (subimages of Envisat ASAR images from http://www.iki.rssi.ru/asp/iw_images/index.html) in the Table.

Рис. 2. Изображения поверхностных проявлений внутренних волн (фрагменты спутниковых снимков с сайта http://www.iki.rssi.ru/asp/iw_images/index.html), перечисленные в таблице.

Parameters of IW packets detected on satellite radar images of the Baltic Sea in 2009—2010

Параметры цугов внутренних волн по данным радиолокационных спутниковых снимков Балтийского моря в 2009—2010 гг.

Date and time, UTC	Coordinates of the center of the packet	Maximal wavelength, m	Front length of the leading wave, m	Number of waves in the packet
05.08.2009 09:03:28	54°57′57″ N 15°46′58″ E	900	23 500	4
01.07.2009 09:03:50	60°47′34″ N 18°15′26″ E	875	16 500	3
12.07.2010 20:13	60°48′10″ N 18°33′50″ E	300	14 600	3
24.07.2010 09:07	59°58′53″ N 19°43'00″ E	900	15 800	4
30.07.2010 09:20	62°49′00″ N 20°06′11″ E	760	15 500	3

tides. The number of waves in the trains was usually ≤10, the maximal wavelength did not exceed 1 km, and the length of the leading wave front was less than 25 km. In July 2010 surface manifestations of IWs were periodically detected in the southern part of the Gulf of Bothnia and to the north and north-west of the island of Gotland.

In situ measurements in the Baltic Sea show fluctuations in current velocities and motions of isotherms on different timescales [10]. Motions with periods of 1—30 min have been observed in the Kiel Bight, while

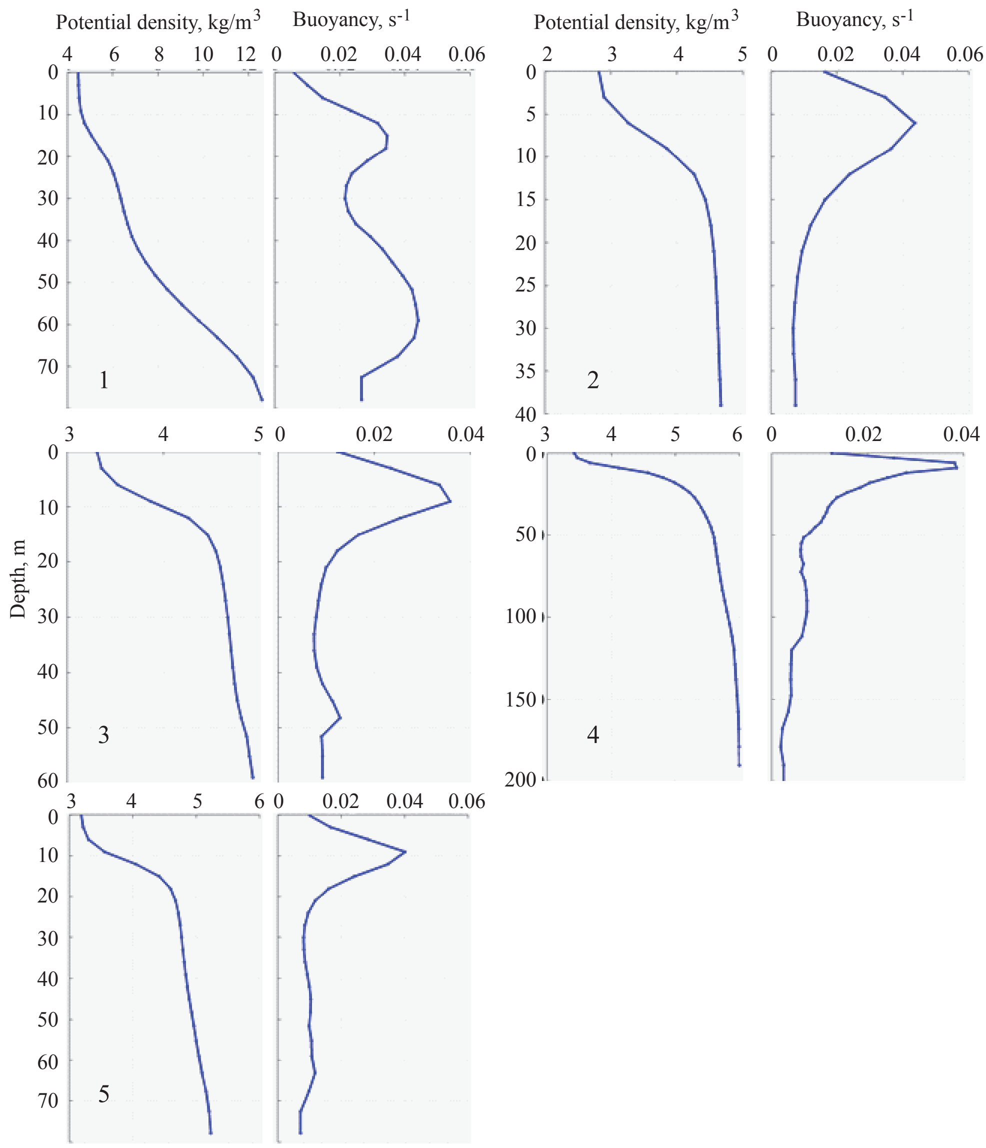

Fig. 3. Density and buoyancy profiles for the locations in the Table.

Рис. 3. Вертикальные профили плотности и плавучести для точек из таблицы.

periods of 5—6 h have been reported in the Gulf of Finland, the Arcona Basin and the Darss Sill area. The resulting temperature (fig. 4) and velocity (fig. 5) variations can be quite large. The largest changes are usually found at the pycnocline location. When there is both a thermocline and a halocline, two IW structures can be observed in the resulting three-layer medium.

Cyclones providing winds of 10—15 m/s in the Baltic Sea cause the generation of IWs with amplitudes of 11—15 m. The associated current velocities are about 11—15 cm/s in the upper layer and about 5—8 cm/s in the lower layer [22]. The characteristics of IWs and internal seiches measured in the Baltic Sea are given in [23, 24]. In particular, IWs with periods of 0.1—1 h observed in the central part of the Gotland Deep formed IW trains with duration of several hours and current amplitudes of about 3 cm/s. IWs in the inertial frequency range can induce currents reaching 20 cm/s.

All available data on IWs in the Baltic Sea were collected, systematized and structured. The resulting database includes: graphical display of information, the possibility of replenishing data and searching by various parameters, export of bibliographic data. The database is integrated into an electronic atlas. The atlas and the database are available on request from the authors of the article. The description can also be found on the web-site of their laboratory in the Nizhny Novgorod State Technical University: http://lmnad.nntu.ru/en/projects/igwresearch/, http://lmnad.nntu.ru/ru/projects/igwatlas_online/.

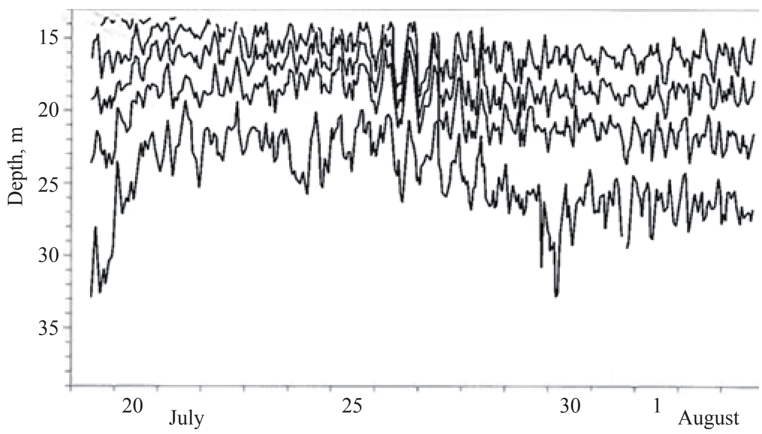

Fig. 4. Isotherms for a 16-day period in July—August 1978 in the Sea of Bothnia [10].

Рис. 4. Изотермы для 16-дневного интервала в июле—августе 1978 г. в Ботническом заливе из [10].

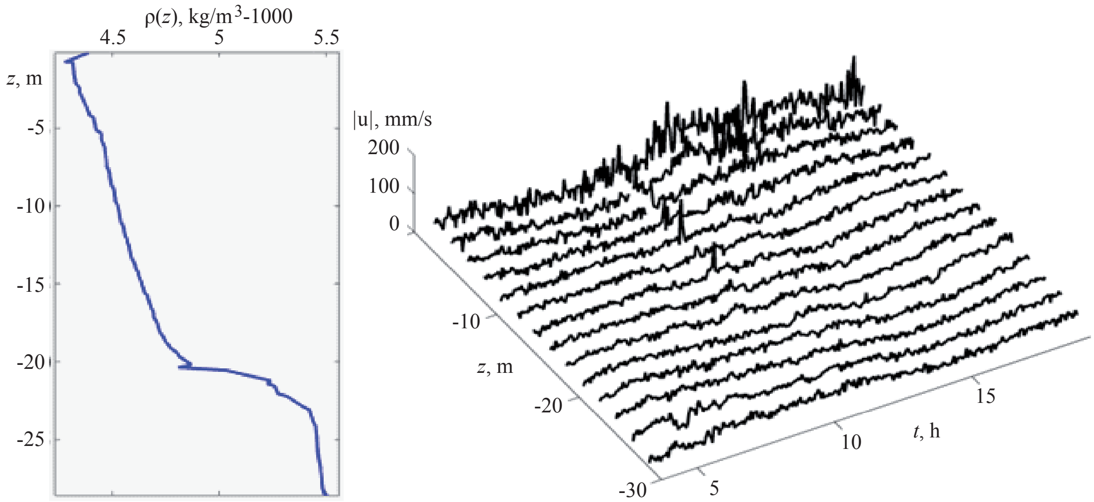

Fig. 5. Density profile (left) and records of the absolute value of horizontal velocity (right) on 16 June 2010 near the coast of the Curonian Spit in the south-eastern part of the Baltic Sea [25].

Рис. 5. Профиль плотности (слева) и записи абсолютных значений горизонтальной скорости (справа) 16 июня 2010 г вблизи Куршской косы в юго-восточной части Балтийского моря [25].

2. Evolution of solitary internal waves in the Baltic Sea. As the Baltic Sea is an almost nontidal sea and does not host major jet-like currents with substantial horizontal or vertical shear [10], the direction of IW propagation depends almost entirely on the source of wave generation (see Section 1) and on refractive properties of the seabed. As trajectories of storm cyclones may cross the Baltic Sea in various locations and such cyclones may have virtually any propagation direction, a large variety of excitation areas and paths of IW propagation in the Baltic Sea may exist. Furthermore, each transformation of an ISW within the Gardner model is reversible. Therefore, such transformations can be formally inverted to demonstrate the possibility of the focusing of a wave train into an ISW. An example of the transformation of an IW group with a moderate amplitude along a cross-section in the Baltic Sea with a specific behaviour of the cubic nonlinearity is presented in [26]. This study shows that the effect of the modulation instability of wavepackets may occur in natural conditions and may lead to the appearance of anomalously large internal rogue waves. Long internal breather-like waves are modelled in [27].

To demonstrate the capacity of the existing databases to highlight possible ISW transformations in the Baltic Sea, we present an example of the hydrological data from the the open source digital climatologic atlas Generalized Digital Environment Model GDEM V.3.0 database (see, e.g., [9] for details of its use) for July. The pathways of the IW propagation are chosen so that they contain so-called critical points (in which the coefficients at nonlinear terms of the underlying Gardner's equation change their signs). The modelled waves are thus forced to propagate through such critical points in which certain basic properties of waves are expected to change.

Calculations of the IW propagation were performed for three straight pathways (cross-sections) in the Baltic Sea (fig. 6, see an insert). The initial problem for the variable-coefficient Gardner's equation was solved using an implicit pseudo-spectral numerical scheme with periodic boundary conditions in a moving reference frame using the coordinate $\tilde{s} = \int \frac{dx}{c(x)} - t$, where t is time, x is the horizontal coordinate along the cross-section, and c is the long IW phase speed.

An example of variable environmental parameters for the modelling of IWs along cross-section 1 (fig. 6) is demonstrated in figs. 7 (see an insert) and 8. This cross-section with a length of approximately 100 km has depths in the range 50—125 m. The pathway of wave propagation is characterized by a gradual increase in the depth after a 25 km long almost horizontal segment where the depth slowly decreases. The linear amplification factor Q (equivalent to the shoaling coefficient for surface waves) is set to 1 at the beginning of the cross-section. This quantity follows the changes in the water deph along the cross-secion. It first increases by a small amount and after $x = 25$ km decreases to 0.4. The linear parameters of IWs (equivalently, the coefficients at the linear terms of Gardner's equation), long wave phase speed c and dispersion coefficient β, correlate well with the depth. They both increase after $x = 25$ km.

The nonlinear parameters α and α_1 (coefficients at the quadratic and the cubic term of Gardner's equation, respectively) determine the nature of solitonic waves in the framework of Gardner's equation. These waves are very sensitive with respect to the shape of density stratification and therefore have a complex behaviour along the chosen cross-sections. The coefficient at the quadratic term α is negative at the beginning of the cross-section and is almost constant about -1×10^{-2} 1/s up to $x = 38$ km. Further on its absolute value rapidly decreases. The coefficient α vanishes and changes the sign around $x = 43$ km and stays then positive with the values of about 1×10^{-2} 1/s. The cubic nonlinearity parameter α_1 is positive along a small segment at the beginning of the cross-section, then it almost linearly decreases, changes the sign around $x = 4$ km and stays negative thereafter. It has a few local maximums and minimums along the cross-section.

As $\alpha < 0$ and $\alpha_1 > 0$ at the initial point of the cross-section, two families of solitons are possible at the beginning of this pathway [28]. One of these families has a polarity corresponding to the sign of α. Solitons of the other family have the opposite polarity and also a lower limit in the amplitude. They must have

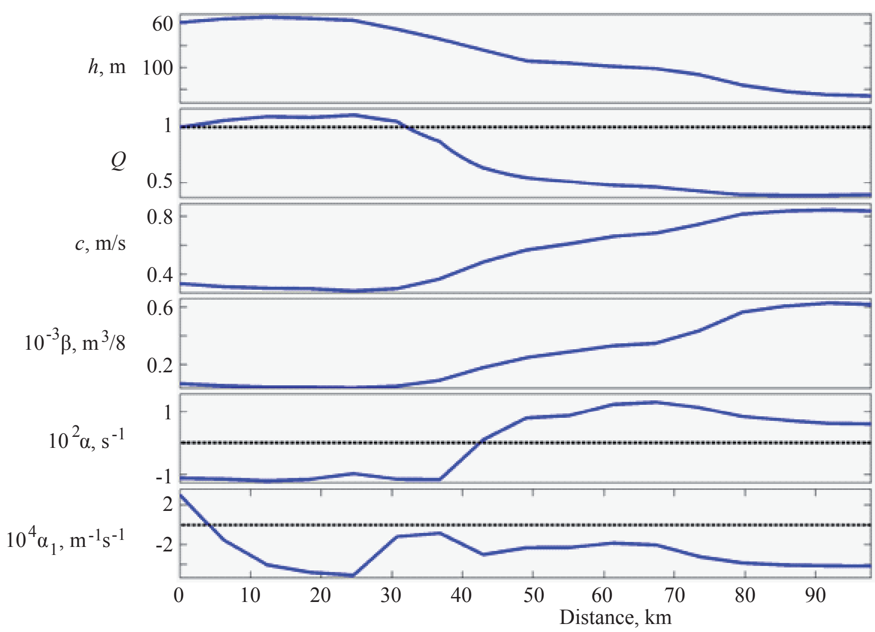

Fig. 8. Coefficients of the variable-coefficient Gardner's equation along cross-section 1.

Рис. 8. Коэффициенты обобщенного уравнения Гарднера вдоль разреза 1.

the amplitude $a \geq 166$ m. This is physically inappropriate for the Baltic Sea conditions and thus only solitons with the negative polarity may exist in this part of the cross-section. For this reason we choose a single-soliton solution of Gardner's equation of negative polarity with the amplitude of $a = -7$ m as the initial condition for the simulations.

The transformation of this structure (in terms of the isopycnal displacement at the maximum of the linear vertical mode) is shown in figs. 9 and 10 in the reference frame $(\widetilde{s}, x)$. The soliton insignificantly changes until the point where the coefficient at the quadratic term of Gardner's equation vanishes ($\alpha = 0$), i.e. over almost 40 km. Further on it transforms into a dispersive oscillating wave tail that propagates over a depression of the isopycnals (so-called negative pedestal). The amplitude of the wave field gradually decreases after

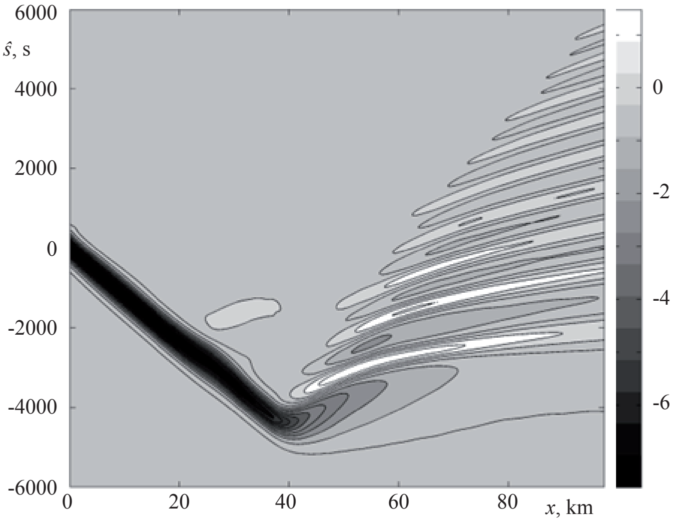

Fig. 9. Contour plot in the space-time domain of an ISW along cross-section 1. The displacement (m) of an isopycnal at the position of maximum of the first vertical linear baroclinic mode is shown.

Рис. 9. Пространственно-временная диаграмма трансформации уединенной внутренней волны вдоль разреза 1. Цветом показано смещение (м) изопикнической поверхности в точке максимума линейной бароклинной моды.

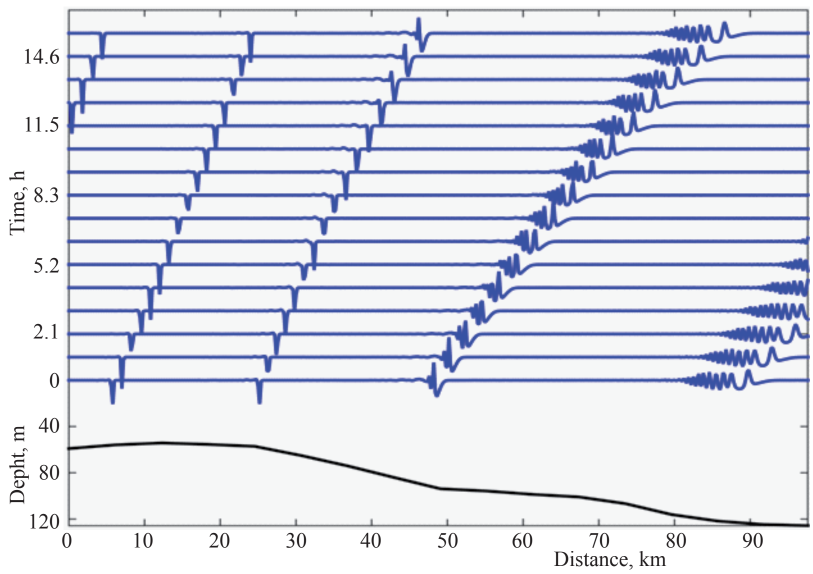

Fig. 10. Transformation of an ISW along cross-section 1. The displacement (m) of an isopycnal at the position of maximum of the first vertical linear baroclinic mode is shown. The vertical shift between successive curves is 5 m.

Рис. 10. Трансформация уединенной внутренней волны вдоль разреза 1. Показано смещение (м) изопикнической поверхности в точке максимума линейной бароклинной моды. Сдвиг по вертикали между последовательными линиями составляет 5 м.

the transformation from the initial value of 7 m to about 1 m at the end of the cross-section. In terms of IW-induced near-surface currents these amplitudes correspond to velocities of 0.15 and 0.04 m/s, respectively. The velocities of the near-surface currents can be used, for example, in studies of surfactant dynamics in a field of IWs, for estimations of hydrodynamic contrasts of the sea surface, etc. The presence of such surface velocity pattern also generates the effects used by the remote sensing to detect the IW surface manifestations.

Spatial variations in the hydrological parameters and the associated variations in the coefficients of the generalized (variable-coefficient) Gardner's equation for cross-sections 2 and 3 are shown in figs. 11 (see an insert), 12 and 13 (see an insert), 14, respectively. The results of simulations of the IW propagation along these

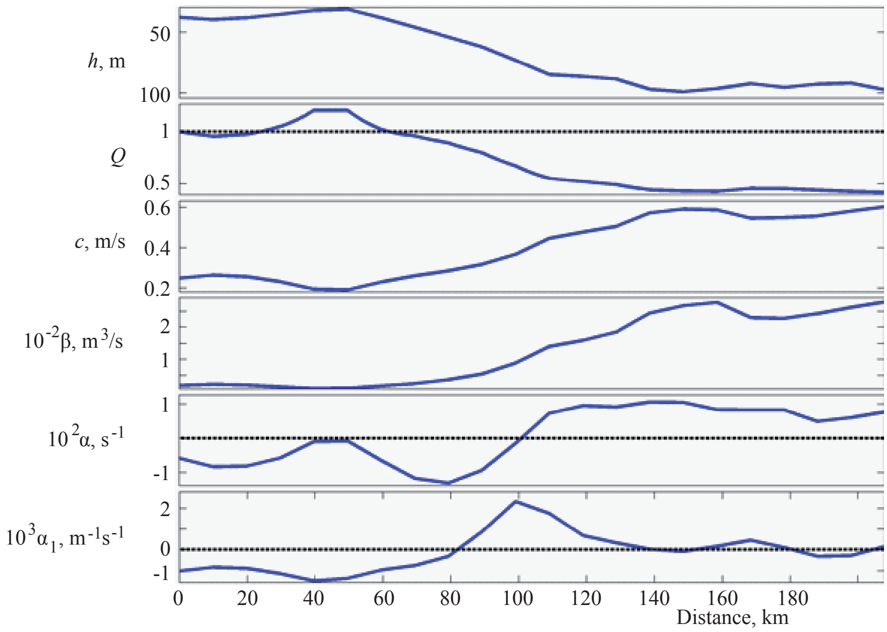

Fig. 12. Coefficients of the variable-coefficient Gardner's equation along cross-section 2.

Рис. 12. Коэффициенты обобщенного уравнения Гарднера вдоль разреза 2.

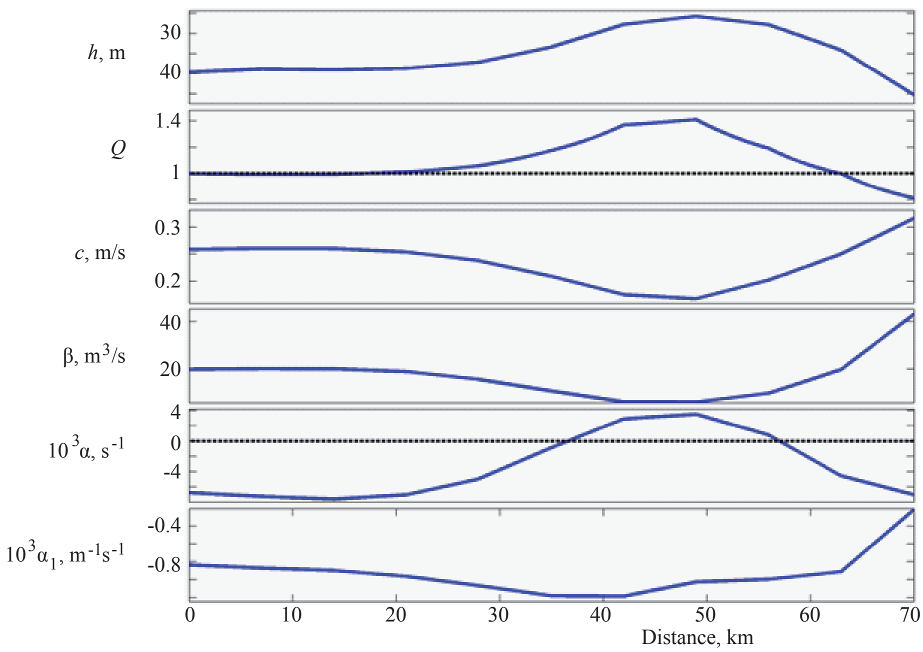

Fig. 14. Coefficients of the generalized Gardner's equation along cross-section 3.

Рис. 14. Коэффициенты обобщенного уравнения Гарднера вдоль разреза 3.

pathways are shown in figs. 15, 16 and 17, 18. The results demonstrate several aspects of complex behaviour of IWs, including adiabatic adjustment, wave amplification, transformation of a wide solitary wave into a sequence of narrow ones, change in the wave polarity, appearance of multiple breather-like disturbances and radiation of oscillatory dispersive tails (wave packets) of small amplitude. All these nonstationary and nonlinear effects develop due to the inhomogeneity of the environment and lead to the formation of a complex field of currents induced by IWs. The properties of such currents can be easily reconstructed and further used as an input

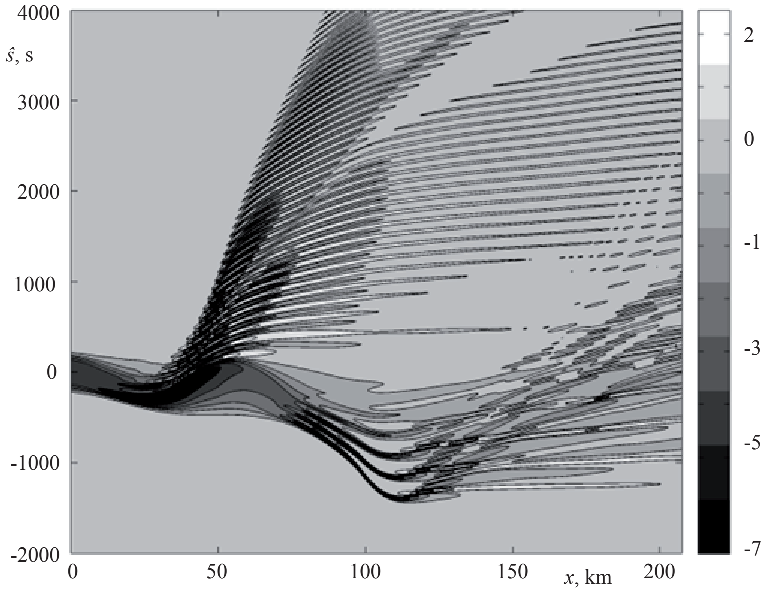

Fig. 15. Contour plot in the space-time domain of an ISW along cross-section 2. The displacement (m) of an isopycnal at the position of maximum of the first vertical linear baroclinic mode is shown.

Рис. 15. Пространственно-временная диаграмма трансформации уединенной внутренней волны вдоль разреза 2. Цветом показано смещение (м) изопикнической поверхности в точке максимума линейной бароклинной моды.

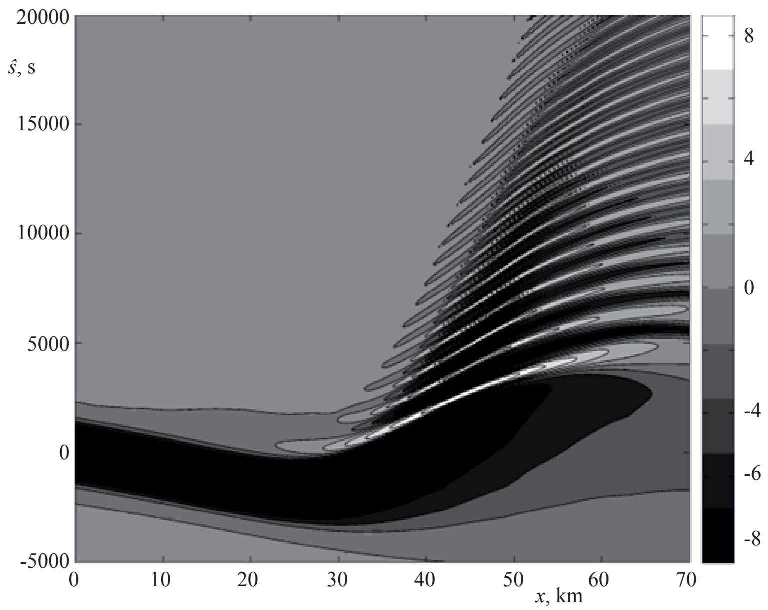

Fig. 16. Contour plot in the space-time domain of an ISW along cross-section 3. The displacement (m) of an isopycnal at the position of maximum of the first vertical linear baroclinic mode is shown.

Рис. 16. Пространственно-временная диаграмма трансформации уединенной внутренней волны вдоль разреза 3. Цветом показано смещение (м) изопикнической поверхности в точке максимума линейной бароклинной моды.

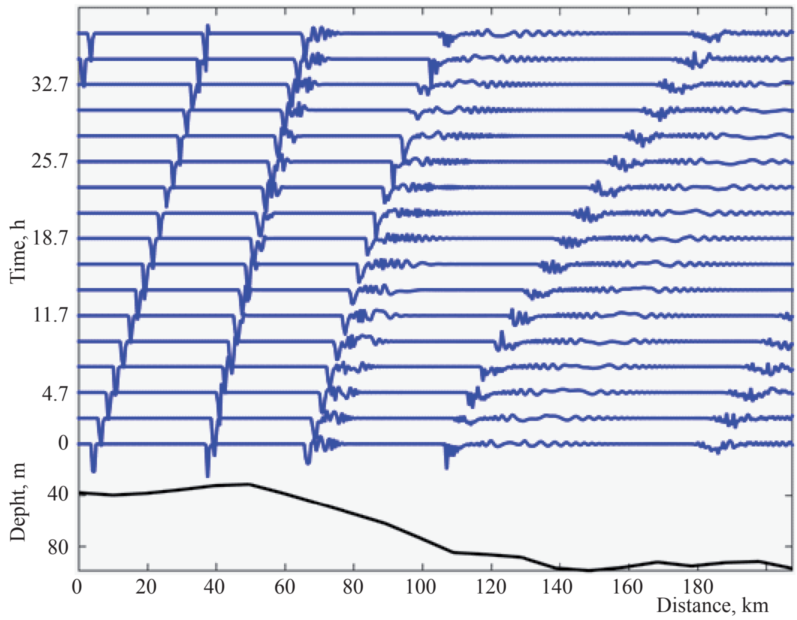

Fig. 17. Transformation of an ISW along cross-section 2. The displacement (m) of an isopycnal at the position of maximum of the first vertical linear baroclinic mode is shown.The vertical shift between successive curves is 5 m.

Рис. 17. Трансформация уединенной внутренней волны вдоль разреза 2. Показано смещение (м) изопикнической поверхности в точке максимума линейной бароклинной моды. Сдвиг по вертикали между последовательными линиями составляет 5 м.

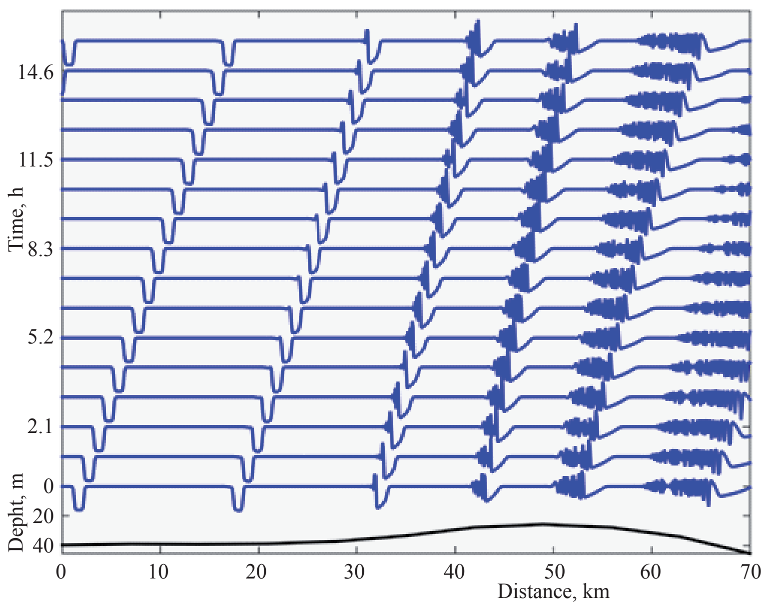

Fig. 18. Transformation of an ISW along cross-section 3. The displacement (m) of an isopycnal at the position of maximum of the first vertical linear baroclinic mode is shown.The vertical shift between successive curves is 10 m.

Рис. 18. Трансформация уединенной внутренней волны вдоль разреза 3. Показано смещение (м) изопикнической поверхности в точке максимума линейной бароклинной моды. Сдвиг по вертикали между последовательными линиями составляет 10 м.

for estimates of wave-induce loads on different structures, models of the near-bottom boundary layer, estimates of seabed erosion and accretion as well as for advective-diffusive models of pollution dynamics.

Conclusions. The theoretically predicted variety of processes of solitary IW propagation and transformations along pathways that include critical points (in which the basic wave properties must change) is reviewed and illustrated for several cross-sections of the Baltic Sea proper. The simulations employ a relatively simple, basically weakly nonlinear model based on Gardner's equation. The pool of existing results, reviewed in this paper, suggests that this model is robust and the outcome of the relevant simulations adequately matches the main properties of IWs in various shelf seas.

Most importantly, this model is able to reproduce several nontrivial kinds of transformations of internal waves in stongly stratified and horizontally inhomogeneous basins such as the Baltic Sea. Different scenarios of wave metamorphoses are illustrated by means of running this model for several transects in the Baltic Sea proper. The results exemplify various phenomena, such as adiabatic adjustment, wave amplification, transformation of a wide solitary wave into a sequence of narrow ones, change in the wave polarity, development of breather-like disturbances, and radiation of oscillatory dispersive wave packets (tails) of small amplitude.

The simulation results show the possibility of transformation of idealized initial IW disturbances with spatial scales or wavelengths matching the values extracted from remote sensing images (500—1000 m, table). Therefore, this model can be used to explicitly interpret satellite and contact observations of IWs if in situ hydrological data and wave records will be provided. As the model is reversible, it may also provide an option to detect the presence of critical points of wave propagation.

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References

1. *Alenius P., Myrberg K., Nekrasov A.* The physical oceanography of the Gulf of Finland: a review. *Boreal Environ. Res.* 1998, 3, 97—125.
2. *Mityagina M. I., Lavrova O. Yu.* Satellite observations of surface manifestations of internal waves in non-tidal seas. *Modern problems of remote sensing of the Earth from space.* 2010, 7, 1, 260—272 (in Russian).
3. *Mityagina M. I., Lavrova O. Yu.* Satellite observations of vortex and wave processes in coastal zone of north-eastern part of the Black Sea. *Issledovanie Zemli iz Kosmosa.* 2009, 5, 72—79 (in Russian).
4. *Lavrova O. Yu., Mityagina M. I., Sabinin K. D.* Manifestations of internal waves on the sea surface in the North-Eastern part of the Black sea. *Issledovanie Zemli iz Kosmosa.* 2009, 6, 49—55 (in Russian).
5. *Lavrova O. Yu., Karimova S. S., Mityagina M. I., Bocharova T. Yu.* On-line satellite monitoring of the aquatories of the Black, Baltic and Caspian Seas in 2009—2010. *Modern Problems of Remote Sensing of the Earth from Space.* 2010, 7, 1, 168—185 (in Russian).
6. *Kurkina O., Pelinovsky E., Talipova T., Soomere T.* Mapping the internal wave field in the Baltic Sea in the context of sediment transport in shallow water. *J. of Coast. Res.* 2011, Special Issue 64, II, 2042—2047.
7. *Grimshaw R., Talipova T., Pelinovsky E., Kurkina O.* Internal solitary waves: propagation, deformation and disintegration. *Nonlin. Processes Geophys.* 2010, 17, 633—649.
8. *Talipova T. G., Pelinovsky E. N., Kurkin A. A., Kurkina O. E.* Modeling the dynamics of intense internal waves on the shelf. *Izvestiya - Atmosph. Ocean Phys.* 2014, 50, 6, 630—637.
9. *Kurkina O., Rouvinskaya E., Talipova T., Soomere T.* Propagation regimes and populations of internal waves in the Mediterranean Sea basin. *Estuar. Coast. Shelf Sci.* 2017, 185, 44—54.
10. *Leppäranta M., Myrberg K.* Physical oceanography of the Baltic Sea. Praxis, Berlin, Heidelberg, New York, Springer, 2009.
11. Axell L. Wind-driven internal waves and Langmuir circulation in a numerical ocean of the Baltic Sea. *J. Geophys. Res.* 2002, 107 (C1), Art. No. 3204.
12. *Feistel R., Nausch G., Wasmund N.* (Eds.) State and evolution of the Baltic Sea, 1952—2005: a detailed 50-year survey of meteorology and climate, physics, chemistry, biology, and marine environment. *John Wiley & Sons*, Inc. 2008.
13. *Van der Lee E.M., Umlauf L.* Internal wave mixing in the Baltic Sea: near- inertial waves in the absence of tide. *J. Geophys. Res.* 2011, 116, C10016-1-16.
14. *Lilover M.-J.* Tidal currents as estimated from ADCP measurements in „practically non-tidal" Baltic Sea. *Proceedings of the IEEE/OES Baltic 2012 International Symposium "Ocean: Past, Present and Future. Climate Change Research, Ocean Observation & Advanced Technologies for Regional Sustainability,"* May 8—11, *Klaipėda, Lithuania,* IEEE; 2012, 1—4.
15. *Ivanov V. A., Lisichenok A. D., Nemirovsky M. S.* Excitation of short-period internal waves by wind pulsations. *Izvestiya - Atmosph. Ocean Phys.* 1987, 23, 2, 179—185.
16. *Vlasenko V. I., Ivanov V. A., Krasin I. G., Lisichenok A. D.* The generation of intensive short-period internal waves in the frontal zone of a coastal upwelling. *Phys. Oceanogr.* 1998, 9, 3, 155—168.
17. *Ivanov V. A., Lisichenok A. D.* Internal waves in the shelf zone and near the shelf edge in the Black Sea. *Phys. Oceanogr.* 2002, 6, 353—360.

18. *Lavrova O. Yu., Mityagina M. I., Sabinin K. D., Serebryany A. N.* Satellite Observations of Surface Manifestations of Internal Waves in the Caspian Sea. *Izvestiya - Atmosph. Ocean Phys.* 2011, 47, 9, 1119—1126.
19. *Meier H.E.M., Döscher R., Faxén T.* A multiprocessor coupled ice-ocean model for the Baltic Sea: application to salt inflow. *J. Geophys. Res.* 2003, 108(C8), Art. No. 3273.
20. *Meier H.E.M.* Modeling the pathways and ages of inflowing salt- and freshwater in the Baltic Sea. *Estuar. Coast. Shelf Sci.* 2007, 74, 717—734.
21. *Soomere T., Delpeche N., Viikmäe B., Quak E., Meier H.E.M., Döös K.* Patterns of current-induced transport in the surface layer of the Gulf of Finland. *Boreal Environ. Res.* 2011, 16, 1, 1—21.
22. *Chernysheva E. S.* On the modeling of long internal waves. In: Davidan, I.N. et al. (eds.), *Problems of the Studies and Mathematical Modeling of the Baltic Sea Ecosystem. Modeling of the Ecosystem Components* Issue No. 3, *Leningrad, Gidrometeoizdat,* 1987, 50—54.
23. *Kol'chitskii N. N., Monin A. S., Paka V. T.* On internal seiches in the deep Baltic Sea. *Doklady Akademii Nauk.* 1996, 346, 2, 249—255.
24. *Golenko N. N., Mel'nikov V. A.* Estimation of spatio-temporal parameters of the internal wave field in the Southwest Baltic Sea using data by towed probe. Scientific reports of the Russian Geographic Union (Kaliningrad branch), *Russian State Univ., Kaliningrad,* 2007, 5, C1—C4. (in Russian).
25. *Kurkina O. E., Kurkin A. A., Dorokhov D. V., Gorbatsky V. V., Morozov E. G., Pankratov A. S.* Distribution, vertical structure and seasonal variability of horizontal currents near the Curonian Spit in south-eastern Baltic Sea in 2010. *Book of Abstracts of 8th Baltic Sea Science Congress* (22—26, August 2011, St. Petersburg, Russia); 2011b, 272.
26. *Talipova T. G., Pelinovsky E. N., Kharif Ch.* Modulation instability of long internal waves with moderate amplitudes in a stratified horizontally inhomogeneous ocean. *JETP Lett.* 2011, 94, 3, 182—186.
27. *Rouvinskaya E., Talipova T., Kurkina O., Soomere T., Tyugin D.* Transformation of internal breathers in the idealised shelf sea conditions. *Cont. Shelf Res.* 2015, 110, 60—71.
28. *Pelinovsky E., Polukhina O., Slunyaev A., Talipova T.* Internal solitary waves. *Solitary Waves in Fluids*. Chapter 4. *WIT Press, Southampton, Boston,* 2007, 85—110.

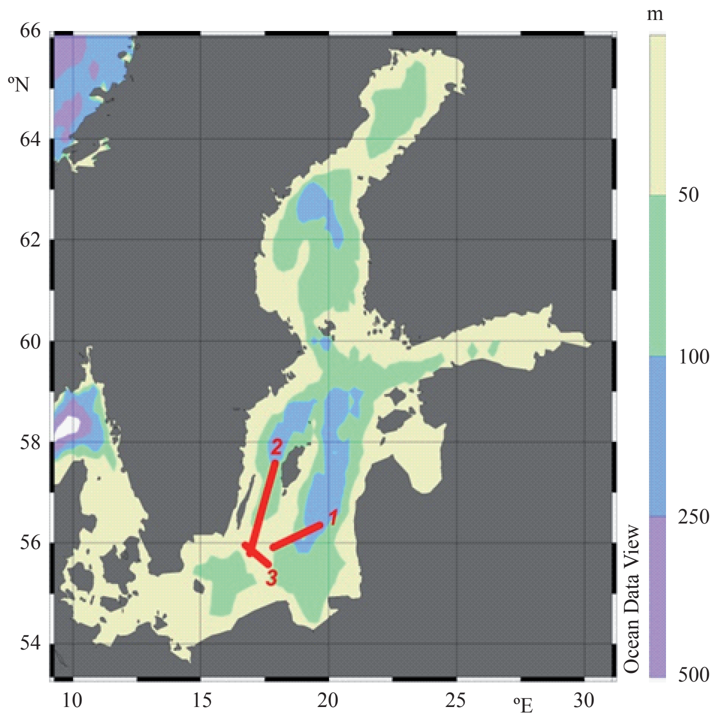

Fig. 6. Bathymetry of the Baltic Sea, with the chosen cross-sections.

Рис. 6. Батиметрическая карта Балтийского моря с выбранными разрезами.

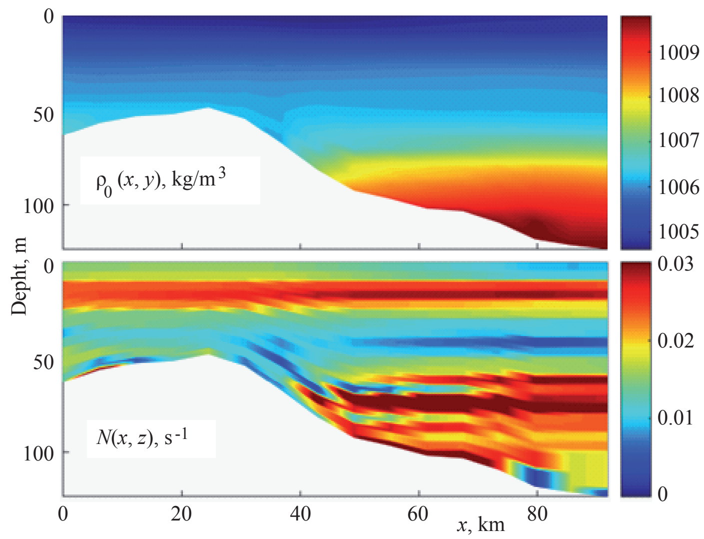

Fig. 7. Undisturbed sea water density and Brunt–Väisälä frequency along cross-section 1.

Рис. 7. Невозмущенное поле плотности и частоты Брента-Вяйсяля вдоль разреза 1.

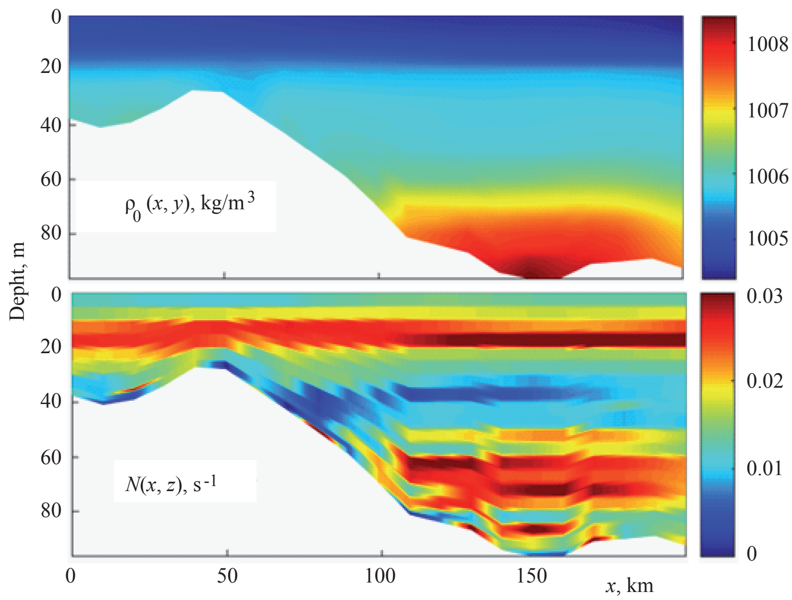

Fig. 11. Undisturbed sea water density and Brunt–Väisälä frequency along cross-section 2.

Рис. 11. Невозмущенное поле плотности и частоты Брента-Вяйсяля вдоль разреза 2.

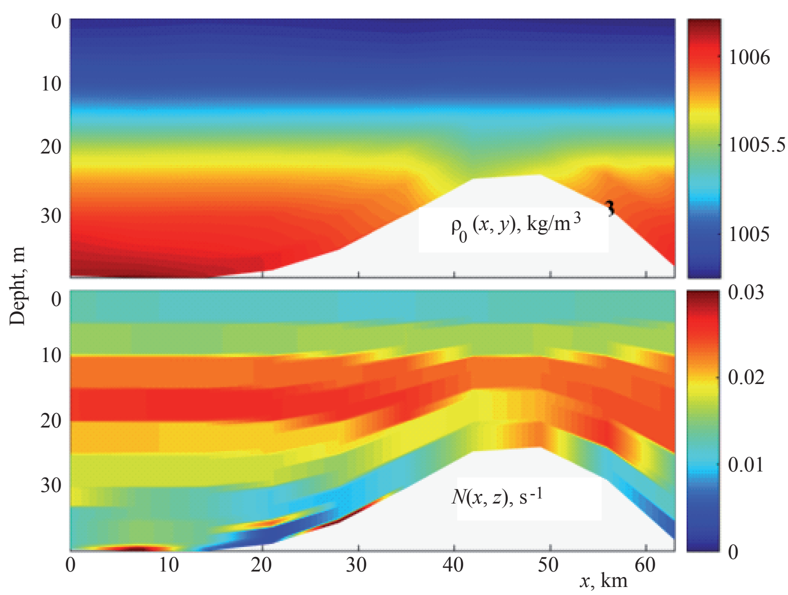

Fig. 13. Undisturbed sea water density and Brunt–Väisälä frequency along cross-section 3.

Рис. 13. Невозмущенное поле плотности и частоты Брента-Вяйсяля вдоль разреза 3.