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Развитие солибора в длинноволновых моделях нелинейных внутренних волн

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Рассматривается генерация солибора внутренней волны из начальных синусоидального и импульсного возмущений в рамках уравнения Гарднера с разными знаками кубической нелинейности и положительным знаком квадратичной нелинейности. В зависимости от типа возмущения и знака кубической нелинейности реализуются разные сценарии. При начальной синусоидальной волне и отрицательном кубе генерируются два ондулярных бора в двух точках обрушения синусоидальной волны. Показан процесс образования толстого солитона из развивающегося ондулярного бора. При положительной кубической нелинейности в точках обрушения, находящихся у креста и подошвы внутренней волны генерируются солитоны положительной и отрицательной полярностей соответственно. Показано, что нелинейное взаимодействие двух солиборов приводит к результирующей отрицательной скорости солитона с малой амплитудой. Если начальное возмущение имеет форму импульса отрицательной полярности, то из него генерируются как солиборы, так и бризеры внутренних волн, и динамика становится достаточно сложной.

Ключевые слова: асимптотические модели; солиборы внутренних волн; бризеры внутренних волн

Solibore Generation in the Nonlinear Long Internal Wave Models

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The internal solibore generation from the initial monochromatic wave or impulse disturbance is studied in the frames of the asymptotic model based on the Gardner equation with both signs of the cubic nonlinear term and positive quadratic nonlinear term. Due to kind of the initial wave and sign of cubic nonlinearity various sec-

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ondary waves are realized. Initial sine wave generates two undular bores in both breaking points when the cubic term is negative. The process of formation of the thick or table solitary wave is scrutinized. When the cubic nonlinearity is positive solitary waves of positive and negative polarities are generated in the breaking points near sine wave crest and trough accordingly. It is shown that interaction of two solibores leads to negative resulting speed of the solitary wave with the lowest amplitude. When the initial wave is the pulse of negative polarity and the cubic nonlinearity is positive the both internal solitary waves and internal breathers are generated and the wave dynamics becomes very complicated
Keywords: asymptotic models; internal wave soliton; internal breather

1. Introduction

Internal tidal wave entering shallow waters transforms into an undular bore. Undular bores are very often observed in the stratified ocean as the vertical displacements of the pycnocline, which lies in the depth of 50-200 m, and they manifest themselves on the ocean surface as the slicks of various intensities, see for instance, (Jackson and Apel, 2004). Sometimes internal undular bore is called solibore. One of examples of solibore evolution in the Arctic Ocean is analyzed in (Talipova et al., 2015). From the physical point of view the undular bore appears in the nonlinear dispersive systems if dispersion is weak in comparison with nonlinearity, and may be realized when the initial disturbance is very long. Appropriate model of such a phenomenon is based on the famous Korteweg-de Vries equation and its extensions (the Gardner equation for example, included the next cubic nonlinear term).

2. Theoretical model and results

We use the canonical form of the Gardner equation

$$\frac{\partial \eta}{\partial t} + 6\eta(1+q\eta)\frac{\partial \eta}{\partial x} + \frac{\partial^3 \eta}{\partial x^3} = 0. \tag{1}$$

with $q = \pm 1$. The sign of quadratic nonlinear term as well as the real values of the coefficients of the Gardner equation in context of the various physical applications are not significant and can be eliminated using appropriate scaling. We compare our numerical results with an analytical rigorous Riemann solution of the dispersionless Gardner equation

$$\frac{\partial \eta}{\partial t} + 6\eta(1+q\eta)\frac{\partial \eta}{\partial x} = 0. \tag{2}$$

given by the expression

$$\eta(x,t) = F(x - V(\eta)t), \qquad \text{where} \quad V(\eta) = 6\eta(1+q\eta). \tag{3}$$

For the sinusoidal initial wave which amplitude A is large enough (A = 1), and case q = -1 two breaking points are predicted within the dispersionless Gardner equation (1) (Kartashova et al, 2013), and also within the full nonlinear theory for two-layer shallow water (Zahibo et al 2007). The second breaking point appears on the back-slope of sine wave significantly later than the first breaking point (t = 10 on Fig.1. left). The first undular bore begins to interact with its own tail due to periodicity even earlier than the second undular bore is formed. The generation of the table-top soliton takes place between both breaking points. The x - t diagramm of the process presented on Fig1 (right) shows that solitary waves of small amplitudes get the additional negative shift when interact with solitary waves of large amplitudes.

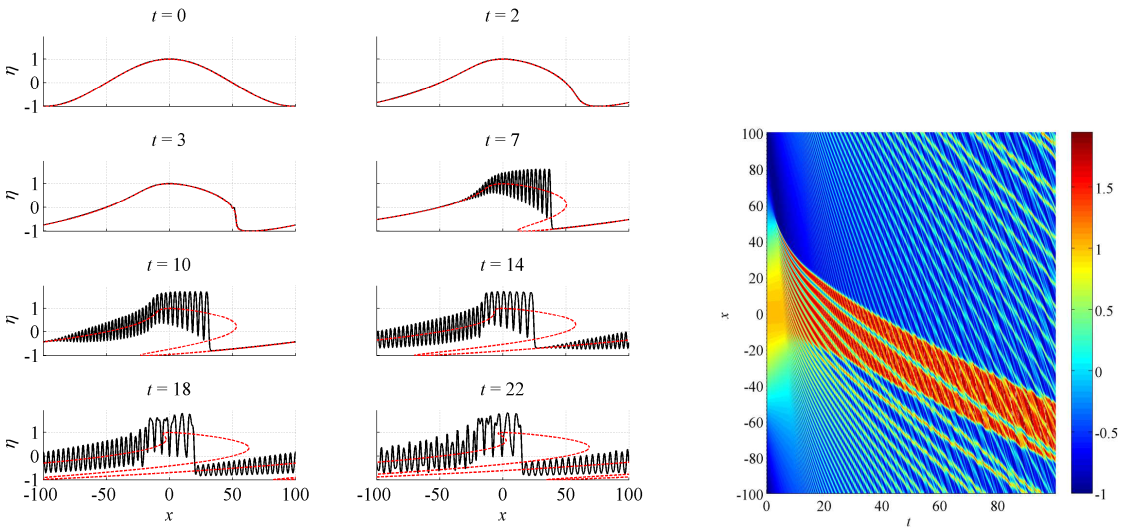

Fig. 1. *Left hand side: snapshots of wave evolution at $A = 1$ ($q = -1$) for the Gardner equation (1) – black line, and purely nonlinear wave deformation ( ) – red line; right hand side: space-time diagram of wave evolution*

The process of solibore development from the initial sinusoidal wave with $A = 2$ and case $q = 1$ is shown on Fig.2. The second breaking point in the sine trough appears later than the first one also and the interaction between the positive solitary waves with the negative ones leads to appearance of the waves with abnormal heights.

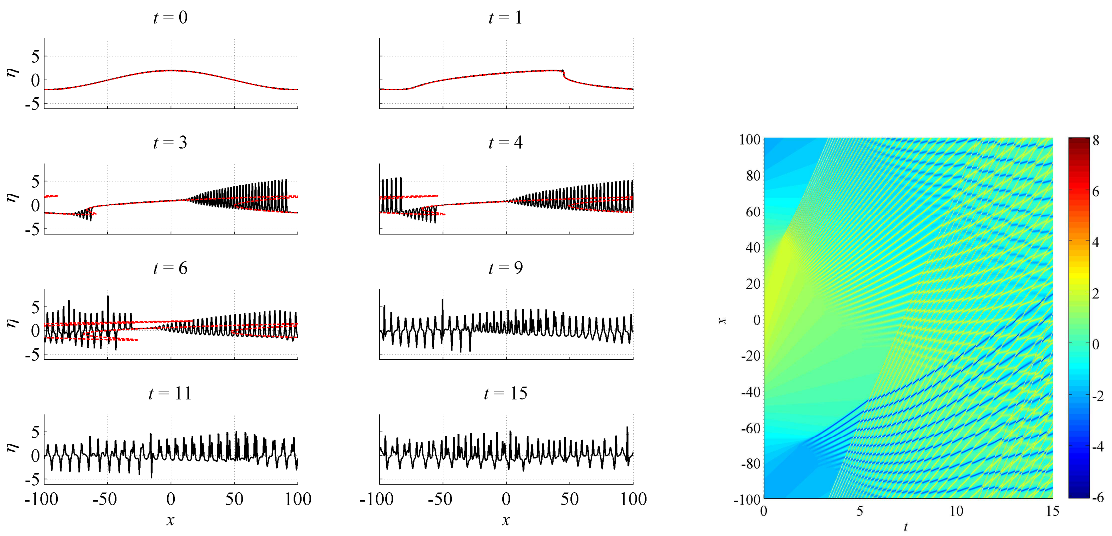

Fig. 2. *Left hand side: snapshots of the wave evolution, $A = 2$, $q = +1$ for the Gardner equation (1) – black line, and purely nonlinear wave deformation (17) – dashed red line; right hand side: space-time diagram of wave evolution*

The initial negative pulse with small amplitude in the frames of the Gardner equation with q =1 generates the nonlinear Airy function, but with amplitude grows it may born the solitary waves as well as the breather like waves what is confirmed by the solution of the inverse problem. Process of the solitary waves and breather-like waves appearance for Gaussian pulse $\eta(x,t=0) = -A\,\mathrm{sech}^2\left(\frac{x}{L}\right)$ with $A = 1$ and $L = 61$ is shown on Fig.3.

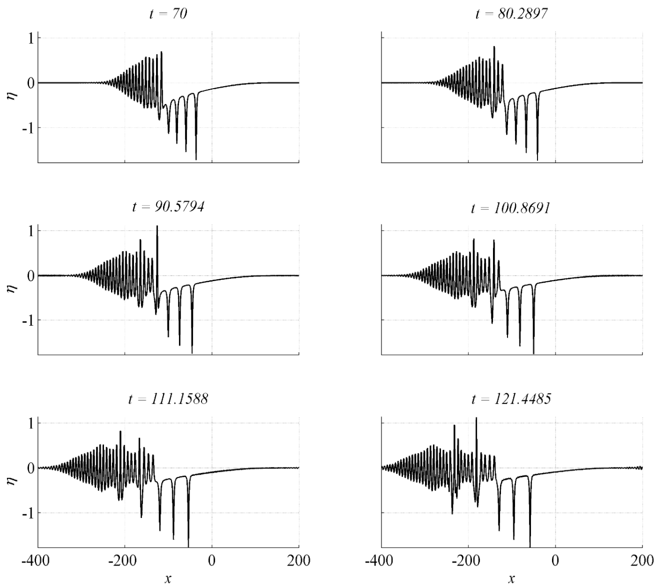

Fig. 3. Snapshots of the negative pulse evolution, A = 1, L = 61 in the Gardner equation with q = +1

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